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R, fuzzy **R**, and Algebraic Kripke-style Semantics* **

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【Abstract】This paper deals with Kripke-style semantics for **FR**, a fuzzy version of **R** of Relevance. For this, first, we introduce **FR**, define the corresponding algebraic structures FR-algebras, and give algebraic completeness results for it. We next introduce an algebraic Kripke-style semantics for **FR**, and connect it with algebraic semantics. We furthermore show that such semantics does not work for **R**.

【Key Words】Kripke-style semantics, Algebraic semantics, Many-valued logic, Fuzzy logic, **R**, **FR**.

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1. Introduction

It is well known that many relevance logicians have had difficulties in providing binary relational Kripke-style semantics, i.e., semantics with binary accessibility relations, for relevance logics (see e.g. [3, 4]). To the best of my knowledge, any satisfactory such semantics for **R** has not yet been introduced. In this paper we show that such semantics can be provided for a fuzzy version of the system **R** of Relevance, although not **R** itself.

Actually, this is a free continuation of the paper [11]. In it the author provided algebraic Kripke-style semantics for Uninorm logic **UL**. Here we introduce algebraic Kripke-style semantics for **FR**, a fuzzy version of **R**.[1] For this, first, in Section 2 we introduce **FR**, define the corresponding algebraic structures FR-algebras, and give algebraic completeness results for it. In Section 3 we introduce an algebraic Kripke-style semantics for **FR**, and connect them with algebraic semantics. We furthermore show that this semantics does not work for **R** (see Example 3.9).

For convenience, we shall adopt the notation and terminology similar to those in [5, 7, 8, 10], and assume familiarity with them (together with the results found in them).

2. The logic **FR** and its algebraic semantics

We base **FR** on a countable propositional language with

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[1] To see why algebraic Kripke-style semantics are interesting, see [12].

formulas *FOR* built inductively as usual from a set of propositional variables *VAR*, binary connectives $\rightarrow$, &, $\wedge$, $\vee$, and constants **f**, **t**, with defined connectives:[2)]

df1. $\sim\phi := \phi \rightarrow \mathbf{f}$
df2. $\phi \leftrightarrow \psi := (\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi)$.

We moreover define $\phi_{\mathbf{t}} := \phi \wedge \mathbf{t}$. For the remainder we shall follow the customary notation and terminology. We use the axiom systems to provide a consequence relation.

We start with the following axiomatization of **FR**.

Definition 2.1 **FR** consists of the following axiom schemes and rules:[3)]

A1. $\phi \rightarrow \phi$ (self-implication, SI)
A2. $(\phi \wedge \psi) \rightarrow \phi$, $(\phi \wedge \psi) \rightarrow \psi$ ($\wedge$-elimination, $\wedge$-E)
A3. $((\phi\rightarrow\psi) \wedge (\phi\rightarrow\chi)) \rightarrow (\phi\rightarrow(\psi\wedge\chi))$ ($\wedge$-introduction, $\wedge$-I)
A4. $\phi \rightarrow (\phi \vee \psi)$, $\psi \rightarrow (\phi \vee \psi)$ ($\vee$-introduction, $\vee$-I)
A5. $((\phi\rightarrow\chi) \wedge (\psi\rightarrow\chi)) \rightarrow ((\phi\vee\psi)\rightarrow\chi)$ ($\vee$-elimination, $\vee$-E)
A6. $(\phi\wedge(\psi\vee\chi))\rightarrow((\phi\wedge\psi)\vee(\phi\wedge\chi))$ ($\wedge\vee$-distributivity, $\wedge\vee$-D)
A7. $(\phi \& \psi) \rightarrow (\psi \& \phi)$ (&-commutativity, &-C)
A8. $(\phi \& \mathbf{t}) \leftrightarrow \phi$ (push and pop, PP)

2) Note that while $\wedge$ is the extensional conjunction connective, & is the intensional conjunction one.

3) A6, indeed, is redundant in **FR**. But we introduce this in order to show that **R** is the **FR** omitting A13. Note that the system omitting both A6 and A13 is not **R** (cf see [1, 2, 4]).

A9. $(\phi \to \psi) \to ((\psi \to \chi) \to (\phi \to \chi))$ (suffixing, SF)
A10. $(\phi \to (\psi \to \chi)) \leftrightarrow ((\phi \,\&\, \psi) \to \chi)$ (residuation, RE)
A11. $\phi \to (\phi \,\&\, \phi)$ (contraction, CR)
A12. $\sim\sim\phi \to \phi$ (double negation elimination, DNE)
A13. $(\phi \to \psi)_t \vee (\psi \to \phi)_t$ (**t**-prelinearity, PL_t)
$\phi \to \psi, \phi \vdash \psi$ (modus ponens, mp)
$\phi, \psi \vdash \phi \wedge \psi$ (adjunction, adj).

A13 is the axiom scheme for linearity, and logics being complete w.r.t. linearly ordered (corresponding) algebras are said to be fuzzy logics (see e.g. [3]).

Note that the system **R** is the **FR** omitting A13. Note also that in **R** (and so **FR**), $\phi \to \psi$ can be defined as $\sim(\phi \,\&\, \sim\psi)$ (df3), and $\phi \,\&\, \psi$ as $\sim(\phi \to \sim\psi)$ (df4).

Proposition 2.2 **FR** proves:
(1) $(\phi \,\&\, (\psi \,\&\, \chi)) \leftrightarrow ((\phi \,\&\, \psi) \,\&\, \chi)$ (&-associativity, AS)
(2) $(\phi \wedge \psi) \to (\phi \,\&\, \psi)$
(3) $(\phi \,\&\, (\psi \wedge \chi)) \leftrightarrow ((\phi \,\&\, \psi) \wedge (\phi \,\&\, \chi))$
(4) $(\phi \to (\psi \vee \chi)) \leftrightarrow ((\phi \to \psi) \vee (\phi \to \chi))$
(5) $((\phi \to (\psi \vee \chi)) \wedge (\psi \to \chi)) \to (\phi \to \chi)$.

Proof: The proof for (1) to (3) is easy, just noting that in order to prove (3) we need A13 (cf. see [1]). We prove (4) and (5).

For the proof of (4), first note that in **R**, we can easily prove $(\phi \to (\psi \vee \chi)) \leftrightarrow (\phi \to \sim(\sim\psi \wedge \sim\chi))$ and $(\phi \to \sim(\sim\psi$

$\wedge$ $\sim\chi)) \leftrightarrow ((\phi$ & $(\sim\psi \wedge \sim\chi)) \rightarrow$ **f**). Then, using (3), we can prove $((\phi$ & $(\sim\psi \wedge \sim\chi)) \rightarrow$ **f**$) \leftrightarrow \sim((\phi$ & $\sim\psi) \wedge (\phi$ & $\sim\chi))$, and using de Morgan laws, we get $\sim((\phi$ & $\sim\psi) \wedge (\phi$ & $\sim\chi)) \leftrightarrow \sim(\phi$ & $\sim\psi) \vee \sim(\phi$ & $\sim\chi)$. Hence, by df3, we obtain $(\phi \rightarrow (\psi \vee \chi)) \leftrightarrow ((\phi \rightarrow \psi) \vee (\phi \rightarrow \chi))$, as required.

For the proof of (5), first note that in **R**, we can easily prove $((\phi \rightarrow \psi) \wedge (\psi \rightarrow \chi)) \rightarrow (\phi \rightarrow \chi)$ using (2). Then, since $((\phi \rightarrow \chi) \wedge (\psi \rightarrow \chi)) \rightarrow (\phi \rightarrow \chi)$, we can obtain $(((\phi \rightarrow \psi) \wedge (\psi \rightarrow \chi)) \vee ((\phi \rightarrow \chi) \wedge (\psi \rightarrow \chi))) \rightarrow (\phi \rightarrow \chi)$. Thus, using A6, we get $((\psi \rightarrow \chi) \wedge ((\phi \rightarrow \psi) \vee (\phi \rightarrow \chi))) \rightarrow (\phi \rightarrow \chi)$. Hence, using (4), we can obtain that $((\phi \rightarrow (\psi \vee \chi)) \wedge (\psi \rightarrow \chi)) \rightarrow (\phi \rightarrow \chi)$, as wished. □

Note that **R** does not prove (5) in Proposition 2.2 (see [5]).

In **FR**, **f** can be defined as $\sim$**t**. A *theory* over **FR** is a set T of formulas. A *proof* in a theory T over **FR** is a sequence of formulas whose each member is either an axiom of **FR** or a member of T or follows from some preceding members of the sequence using the two rules in Definition 2.1. T $\vdash \phi$, more exactly T $\vdash_{\mathbf{FR}} \phi$, means that ϕ is *provable* in T w.r.t. **FR**, i.e., there is a **FR**-proof of ϕ in T. The relevant deduction theorem ($\mathrm{RDT}_{\mathbf{t}}$) for **FR** is as follows:

Proposition 2.3 ([7]) Let T be a theory, and ϕ, ψ formulas.
($\mathrm{RDT}_{\mathbf{t}}$) T $\cup$ $\{\phi\} \vdash \psi$ iff T $\vdash \phi_{\mathbf{t}} \rightarrow \psi$.

For convenience, "$\sim$", "$\wedge$", "$\vee$", and "$\rightarrow$" are used ambiguously as propositional connectives and as algebraic operators, but context should make their meaning clear.

The algebraic counterpart of **FR** is the class of *FR-algebras*. Let $x_t := x \wedge t$. They are defined as follows.

Definition 2.4 (i) A *pointed commutative residuated distributive lattice* is a structure $\mathbf{A} = (A, t, f, \wedge, \vee, *, \rightarrow)$ such that:

(I) $(A, \wedge, \vee)$ is a distributive lattice.

(II) $(A, *, t)$ is a commutative monoid.

(III) $y \leq x \rightarrow z$ iff $x * y \leq z$, for all $x, y, z \in A$ (residuation).

(ii) (Dunn-algebras, [1, 2]) A *Dunn-algebra* is a pointed commutative residuated distributive lattice satisfying:

(IV) $x \leq x * x$ (contraction).

(V) $(x \rightarrow f) \rightarrow f \leq x$ (double negation elimination).

(iii) (FR-algebras) A *FR-algebra* is a Dunn-algebra satisfying:

(VI) $t \leq (x \rightarrow y)_t \vee (y \rightarrow x)_t$ (pl_t).

Note that the class of Dunn-algebras characterizes the system **R**. Note also that Dunn-algebras are also called De Morgan monoids.

Additional (unary) negation and (binary) equivalence operations are defined as in Section 2.1: $\sim x := x \rightarrow f$ and $x \leftrightarrow y := (x \rightarrow y) \wedge (y \rightarrow x)$.

The class of all FR-algebras is a variety which will be denoted by FR.

FR-algebra is said to be *linearly ordered* if the ordering of its algebra is linear, i.e., $x \leq y$ or $y \leq x$ (equivalently, $x \wedge y = x$ or $x \wedge y = y$) for each pair x, y.

Definition 2.5 (Evaluation) Let $\mathcal{A}$ be an algebra. An *$\mathcal{A}$-evaluation* is a function $v : FOR \rightarrow \mathcal{A}$ satisfying: $v(\phi \rightarrow \psi) = v(\phi) \rightarrow v(\psi)$, $v(\phi \wedge \psi) = v(\phi) \wedge v(\psi)$, $v(\phi \vee \psi) = v(\phi) \vee v(\psi)$, $v(\phi \,\&\, \psi) = v(\phi) * v(\psi)$, $v(\mathbf{f}) = f$, (and hence $v(\sim\phi) = \sim v(\phi)$ and $v(\mathbf{t}) = t$).

Definition 2.6 Let $\mathcal{A}$ be a FR-algebra, T a theory, ϕ a formula, and K a class of FR-algebras.

(i) (Tautology) ϕ is a *t-tautology* in $\mathcal{A}$, briefly an *$\mathcal{A}$-tautology* (or *$\mathcal{A}$-valid*), if $v(\phi) \geq t$ for each $\mathcal{A}$-evaluation v.

(ii) (Model) An $\mathcal{A}$-evaluation v is an *$\mathcal{A}$-model* of T if $v(\phi) \geq t$ for each $\phi \in T$. By *Mod(T, $\mathcal{A}$)*, we denote the class of $\mathcal{A}$-models of T.

(iii) (Semantic consequence) ϕ is a *semantic consequence* of T w.r.t. K, denoting by $T \vDash_{K} \phi$, if $Mod(T, \mathcal{A}) = Mod(T \cup \{\phi\}, \mathcal{A})$ for each $\mathcal{A} \in K$.

Definition 2.7 (**FR**-algebra) Let $\mathcal{A}$, T, and ϕ be as in Definition 2.6. $\mathcal{A}$ is a ***FR**-algebra* iff whenever ϕ is FR-provable in T (i.e. $T \vdash_{\mathbf{FR}} \phi$), it is a semantic consequence of T w.r.t. the set $\{\mathcal{A}\}$ (i.e. $T \vDash_{\{A\}} \phi$), $\mathcal{A}$ a FR-algebra. By *$MOD^{(l)}$(**FR**)*, we denote the class of (linearly ordered) **FR**-algebras. Finally, we write $T \vDash^{(l)}_{\mathbf{FR}} \phi$ in place of $T \vDash_{MOD^{(l)}(\mathbf{FR})} \phi$.

Note that since each condition for the FR-algebra has a form of equation or can be defined in equation (exercise), it can be ensured that the class of all FR-algebras is a variety.

We first show that classes of provably equivalent formulas form a FR-algebra. Let T be a fixed theory over **FR**. For each formula ϕ, let $[\phi]_T$ be the set of all formulas ψ such that $T \vdash_{\mathbf{FR}} \phi \leftrightarrow \psi$ (formulas T-provably equivalent to ϕ). A_T is the set of all the classes $[\phi]_T$. We define that $[\phi]_T \rightarrow [\psi]_T = [\phi \rightarrow \psi]_T$, $[\phi]_T * [\psi]_T = [\phi \,\&\, \psi]_T$, $[\phi]_T \wedge [\psi]_T = [\phi \wedge \psi]_T$, $[\phi]_T \vee [\psi]_T = [\phi \vee \psi]_T$, $t = [\mathbf{t}]_T$, and $\bot_{\mathbf{f}} = [\mathbf{f}]_T$. By $\boldsymbol{A}_T$, we denote this algebra.

Proposition 2.8 For T a theory over **FR**, $\mathbf{A}_T$ is a **FR**-algebra.

Proof: Note that A1 to A6 ensure that $\wedge$ and $\vee$ satisfy (I) in Definition 2.4; that A7, A8, and AS ensure that & satisfies (II); that A10 ensures that (III) holds; and that A11, A12, and A13 ensure that (IV), (V), and (VI), respectively, hold. It is obvious that $[\phi]_T \leq [\psi]_T$ iff $T \vdash_{\mathbf{FR}} \phi \leftrightarrow (\phi \wedge \psi)$ iff $T \vdash_{\mathbf{FR}} \phi \rightarrow \psi$. Finally recall that $\mathbf{A}_T$ is a **FR**-algebra iff $T \vdash_{\mathbf{FR}} \psi$ implies $T \vDash_{\mathbf{FR}} \psi$, and observe that for ϕ in T, since $T \vdash_{\mathbf{FR}} \mathbf{t} \rightarrow \phi$, it follows that $[\mathbf{t}]_T \leq [\phi]_T$. Thus it is a **FR**-algebra. □

We next note that the nomenclature of the prelinearity condition is explained by the subdirect representation theorem below.

Proposition 2.9 (Cf. [10]) Each FR-algebra is a subdirect product of linearly ordered FR-algebras.

Theorem 2.10 (Strong completeness) Let T be a theory, and ϕ a formula. T $\vdash_{\mathbf{FR}}$ ϕ iff T $\models_{\mathbf{FR}}$ ϕ iff T $\models^{1}_{\mathbf{FR}}$ ϕ.

Proof: (i) T $\vdash_{\mathbf{FR}}$ ϕ iff T $\models_{\mathbf{FR}}$ ϕ. The left-to-right direction follows from definition. The right-to-left direction is as follows: from Proposition 2.8, we obtain $\mathbf{A}_T \in$ MOD(FR), and for $\mathbf{A}_T$-evaluation v defined as $v(\psi) = [\psi]_T$, it holds that v $\in$ Mod(T, $\mathbf{A}_T$). Thus, since from T $\models_{\mathbf{FR}}$ ϕ we obtain that $[\phi]_T = v(\phi) \geq t$, T $\vdash_{\mathbf{FR}}$ $\mathbf{t} \to \phi$. Then, since T $\vdash_{\mathbf{FR}}$ $\mathbf{t}$, by (mp) T $\vdash_{\mathbf{FR}}$ ϕ, as required.

(ii) T $\models_{\mathbf{FR}}$ ϕ iff T $\models^{1}_{\mathbf{FR}}$ ϕ. It follows from Proposition 2.9. □

3. Kripke-style semantics for FR

Here we consider algebraic Kripke-style semantics for **FR**.

Definition 3.1 (Algebraic Kripke frame) An *algebraic Kripke frame* is a structure $\mathbf{X} = (X, t, f, \leq, *, \to)$ such that $(X, t, f, \leq, *, \to)$ is a linearly ordered residuated pointed commutative monoid. The elements of **X** are called *nodes*.

Definition 3.2 (FR frame) A *FR frame* is an algebraic Kripke frame, where $x = (x \to f) \to f$, and $*$ is contractive, i.e., $x \leq x * x$.

An *evaluation* or *forcing* on an algebraic Kripke frame is a relation $\Vdash$ between nodes and propositional variables, and arbitrary formulas subject to the conditions below: for every propositional variable p,

(AHC) if $x \Vdash p$ and $y \leq x$, then $y \Vdash p$; and

for arbitrary formulas,

(t) $x \Vdash \mathbf{t}$ iff $x \leq t$;
(f) $x \Vdash \mathbf{f}$ iff $x \leq f$;
($\wedge$) $x \Vdash \phi \wedge \psi$ = iff $x \Vdash \phi$ and $x \Vdash \psi$;
($\vee$) $x \Vdash \phi \vee \psi$ iff $x \Vdash \phi$ or $x \Vdash \psi$;
(&) $x \Vdash \phi \,\&\, \psi$ iff there are $y, z \in X$ such that $y \Vdash \phi$, $z \Vdash \psi$, and $x \leq y * z$;
($\rightarrow$) $x \Vdash \phi \rightarrow \psi$ iff for all $y \in X$, if $y \Vdash \phi$, then $x * y \Vdash \psi$.

Definition 3.3 (i) (Algebraic Kripke model) An *algebraic Kripke model* is a pair ($\mathbf{X}$, $\Vdash$), where $\mathbf{X}$ is an algebraic Kripke frame and $\Vdash$ is a forcing on $\mathbf{X}$.
(ii) (FR model) A *FR model* is a pair ($\mathbf{X}$, $\Vdash$), where $\mathbf{X}$ is a FR frame and $\Vdash$ is a forcing on $\mathbf{X}$.

Definition 3.4 (Cf. [9]) Given an algebraic Kripke model ($\mathbf{X}$, $\Vdash$), a node x of $\mathbf{X}$ and a formula ϕ, we say that *x forces ϕ* to express $x \Vdash \phi$. We say that ϕ is *true* in ($\mathbf{X}$, $\Vdash$) if $t \Vdash \phi$, and that ϕ is *valid* in the frame **X** (expressed by **X** models ϕ) if ϕ is true in (**X**, $\Vdash$) for every forcing $\Vdash$ on **X**.

For soundness and completeness for **FR**, let $\vdash_{\mathbf{FR}} \phi$ be the theoremhood of ϕ in **FR**. First we note the following lemma.

Lemma 3.5 (Hereditary Lemma, HL) Let **X** be an algebraic Kripke frame. For any sentence ϕ and for all nodes x, y $\in$ **X**, if $x \Vdash \phi$ and $y \leq x$, then $y \Vdash \phi$.

Proof: Easy. □

Proposition 3.6 (Soundness) If $\vdash_{\mathbf{FR}} \phi$, then ϕ is valid in every FR frame.

Proof: We prove the validity of A11 as an example: it suffices to show that if $x \Vdash \phi$, then $x \Vdash \phi \,\&\, \phi$. Assume $x \Vdash \phi$. Then, since $x \leq x * x$, using (&), we can obtain $x \Vdash \phi \,\&\, \phi$, as required.

The proof for the other cases is left to the interested reader. □

By a *chain*, we mean a linearly ordered algebra. The next proposition connects algebraic Kripke semantics and algebraic semantics for **FR** (cf. see [9]).

Proposition 3.7 (i) The {t, f, $\leq$, $*$, $\rightarrow$} reduct of a FR-chain $\mathbf{A}$ is a FR frame.

(ii) Let **X** = (X, t, f, $\leq$, $*$, $\rightarrow$) be a FR frame. Then the structure **A** = (X, t, f, max, min, $*$, $\rightarrow$) is a FR-algebra (where *max* and *min* are meant w.r.t. $\leq$).

(iii) Let **X** be the {t, f, $\leq$, $*$, $\rightarrow$} reduct of a FR-chain $\mathbb{A}$, and let v be an evaluation in $\mathbb{A}$. Let for every atomic formula p and for every x $\in$ $\mathbb{A}$, x $\Vdash$ p iff x $\leq$ v(p). Then (**X**, $\Vdash$) is a FR model, and for every formula ϕ and for every x $\in$ $\mathbb{A}$, we obtain that: x $\Vdash$ ϕ iff x $\leq$ v(ϕ).

Proof: The proof for (i) and (ii) is easy. For the proof of (iii), see Proposition 3.8 in [10]. □

Theorem 3.8 (Strong completeness) **FR** is strongly complete w.r.t. the class of all FR-frames.

Proof: It follows from Proposition 3.7 and Theorem 2.10. □

Let an *R frame* **X** be an FR frame on a partially ordered monoid in place of a linearly ordered monoid, let an evaluation or forcing $\Vdash$ on an R frame be the same as that on a FR frame, and let (**X**, $\Vdash$) be an R model. Then, at first glance, (**X**, $\Vdash$) seems to be a model for **R**. But actually it is not. The following example verifies it.

Example 3.9 An R model (**X**, $\Vdash$) validates Proposition 2.2 (5), i.e., t $\Vdash$ (($\phi \rightarrow (\psi \vee \chi)) \wedge (\psi \rightarrow \chi)) \rightarrow (\phi \rightarrow \chi$).

Proof: By (→) and (∧), we assume $x \vdash (\phi \to (\psi \lor \chi)$ and $x \vdash \psi \to \chi$, and show $x \vdash \phi \to \chi$. For this last, we further assume $y \vdash \phi$ and show $x * y \vdash \chi$. By the suppositions and (→), we have $x * y \vdash \psi \lor \chi$, therefore $x * y \vdash \psi$ or $x * y \vdash \chi$ by (∨). Let $x * y \vdash \psi$. Then, since $x \vdash \psi \to \chi$, by (→) we obtain $x * (x * y) \vdash \chi$, therefore $(x * x) * y \vdash \chi$ by the associativity of *. Then, since $x \leq x * x$, using Lemma 3.5, we get $x * y \vdash \chi$. □

This sentence is not a theorem of **R** but a theorem of **FR**. Thus this model is not for **R**.

4. Concluding remark

We investigated algebraic Kripke-style semantics for **FR**, a fuzzy version of **R**. We proved soundness and completeness theorems. But we did not provide algebraic Kripke-style semantics for **R**. This is an open problem left in this paper.

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R, fuzzy R, and Algebraic Kripke-style Semantics

양 은 석

이 글에서 우리는 연관 논리 **R**을 퍼지화한 체계 **FR**을 위한 크립키형 의미론을 다룬다. 이를 위하여 먼저 **FR** 체계를 소개하고 그에 상응하는 FR-대수를 정의한 후 **FR**이 대수적으로 완전하다는 것을 보인다. 다음으로 **FR**을 위한 대수적 크립키형 의미론을 소개하고 이를 대수적 의미론과 연관 짓는다. 마지막으로 이러한 의미론이 **R**에는 적용될 수 없다는 점을 보인다.

주요어: **R**, **FR**, (대수적) 크립키형 의미론, 대수적 의미론, 다치 논리, 퍼지 논리