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Weakening-free fuzzy logics with the connective Δ (Ⅱ): a variant of Baaz projection*

Eunsuk Yang

【Abstract】Yang [12] investigated weakening-free fuzzy logics expanded by the delta connective Δ, which can be interpreted as Baaz's projection and its generalizations. In this paper, we keep investigating such logics with an alternative delta connective Δ, which can be regarded as a variant of the Baaz projection. The main difference is that although our new Δ satisfies many properties of Baaz projection, it can nether be interpreted as Baaz's projection itself nor its generalizations. For this, we first introduce several weakening-free fuzzy logics with the alternative connective Δ. The algebraic structures corresponding to the systems are then defined, and their algebraic completeness is proved.

【Key Words】(Substructural) Fuzzy logic, Uninorm (based) logic, Baaz projection and its variant.

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1. Introduction

Yang [12] investigated weakening-free fuzzy logics expanded by the delta connective Δ, which can be interpreted as Baaz's projection and its generalizations. In this paper, we keep going the investigation of [12]. More exactly, we investigate such logics expanded by the delta connective Δ, which can be regarded as a variant of Baaz projection. Here, the connective Δ is a variant of Baaz projection in the sense that although it satisfies many properties of Baaz projection, it can neither be interpreted as Baaz's projection nor its generalizations (see Remark 3.6).

As mentioned in [12], in the literature of t-norm logics, not only a number of axiomatic extensions but also expansions of MTL (Monoidal t-norm logic) by means of expanding the language with new connectives have been introduced. Among its expansions, the most well-known examples are the expansions with the connective Δ. The connective Δ corresponds to the so called Baaz's Delta projection of fuzzy logic (see [1]). As is known to us, truth function (denoted also by Δ) of the connective Δ in [1] is defined as follows: letting 1, 0 be top and bottom elements, respectively,

$$\text{(BP)} \qquad \Delta x = \begin{cases} 1 & \text{if } x = 1 \\ 0 & \text{otherwise.} \end{cases}$$

This interpretation, i.e., *Baaz 0-1 projection*, is a very natural one in t-norm logics because t-norms as semantics for these logics have 1 as the sole designated element.

However, uninorms have identity lying anywhere in [0, 1] and so the interpretation is no more natural one in uninorm (based) logics such as UL (Uninorm logic) in that, in general, uninorms as semantics for these logics have elements greater than or equal to identity as designated elements. This raises the interesting question as to whether weakening-free fuzzy logics such as UL have a similar expansion. As its answer, Yang introduced several systems with the connective Δ to be interpreted as the Baaz projection and its generalizations.

One interesting fact to state is that such systems may have the connective Δ to be interpreted not merely as the Baaz projection and its generalizations, but also as its variants. For its verification, here we introduce such logics expanded by an alternative delta connective Δ, which can be regarded as a variant of Baaz projection in the above sense. For this, we first introduce several weakening-free fuzzy systems with the alternative connective Δ. The algebraic structures corresponding to the systems are then defined, and associated algebraic completeness is proved.

For convenience, we shall adopt the notation and terminology similar to those in [2, 3, 4, 6, 8, 12], and assume familiarity with them (together with results found therein).

2. Syntax

We base weakening-free fuzzy systems with the connective Δ on a countable propositional language with formulas *FOR* built inductively as usual from a set of propositional variables *VAR*, unary and binary connectives Δ, $\rightarrow$, $\&$, $\wedge$, $\vee$, and constants **T**, **F**, **f**, **t**, with defined connectives:

df1. $\sim\phi := \phi \rightarrow \mathbf{f}$, and
df2. $\phi \leftrightarrow \psi := (\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi)$.

We may define **t** as $\mathbf{f} \rightarrow \mathbf{f}$. We moreover define ϕ^n_t as $\phi_t \,\&\, \cdots \,\&\, \phi_t$, n factors, where $\phi_t := \phi \wedge \mathbf{t}$. For the remainder we shall follow the customary notation and terminology. We use the axiom systems to provide a consequence relation.

We start with the following axiomatization of $\mathbf{UL}_\Delta$ (**UL** with the connective Δ) as the basic logic.

Definition 2.1 $\mathbf{UL}_\Delta$ consists of the following axiom schemes and rules:

A1. $\phi \rightarrow \phi$ (self-implication, SI)
A2. $(\phi \wedge \psi) \rightarrow \phi$, $(\phi \wedge \psi) \rightarrow \psi$ ($\wedge$-elimination, $\wedge$-E)
A3. $((\phi\rightarrow\psi)\wedge(\phi\rightarrow\chi)) \rightarrow (\phi\rightarrow(\psi\wedge\chi))$ ($\wedge$-introduction, $\wedge$-I)
A4. $\phi \rightarrow (\phi \vee \psi)$, $\psi \rightarrow (\phi \vee \psi)$ ($\vee$-introduction, $\vee$-I)
A5. $((\phi\rightarrow\chi)\wedge(\psi\rightarrow\chi)) \rightarrow ((\phi\vee\psi)\rightarrow\chi)$ ($\vee$-elimination, $\vee$-E)
A6. $\phi \rightarrow \mathbf{T}$ (verum ex quolibet, VE)
A7. $\mathbf{F} \rightarrow \phi$ (ex falso quadlibet, EF)

A8. $(\phi \,\&\, \psi) \to (\psi \,\&\, \phi)$ (&-commutativity, &-C)

A9. $(\phi \,\&\, \mathbf{t}) \leftrightarrow \phi$ (push and pop, PP)

A10. $(\phi \to \psi) \to ((\psi \to \chi) \to (\phi \to \chi))$ (suffixing, SF)

A11. $(\phi \to (\psi \to \chi)) \leftrightarrow ((\phi \,\&\, \psi) \to \chi)$ (residuation, RE)

A12. $(\phi \to \psi)_{\mathbf{t}} \vee (\psi \to \phi)_{\mathbf{t}}$ (**t**-prelinearity, $PL_{\mathbf{t}}$).

A13. $\Delta\phi \to \mathbf{t}$

A14. $\phi_{\mathbf{t}} \to \Delta\phi$

A15. $(\phi \to \mathbf{t}) \to (\phi \to \Delta\phi)$

A16. $\Delta(\phi \vee \psi) \to (\Delta\phi \vee \Delta\psi)$ ($\Delta\vee$-distribution, $\Delta\vee$-D)

A17. $(\Delta\phi \wedge \Delta\psi) \to \Delta(\phi \wedge \psi)$ ($\Delta\wedge$-D)

A18. $\Delta\phi \to \phi$ (Δ-reflexivity, ΔRF)

A19. $\Delta\phi \to \Delta\Delta\phi$ (Δ-transitivity, ΔTRS)

A20. $\Delta(\phi \to \psi) \to (\Delta\phi \to \Delta\psi)$ (Δ-monotonicity, ΔMN)

A21. $\Delta(\phi \,\&\, \phi) \to (\Delta\phi \,\&\, \Delta\phi)$ (Δ&-D)

$\phi \to \psi, \phi \vdash \psi$ (modus ponens, mp)

$\phi, \psi \vdash \phi \wedge \psi$ (adjunction, adj).

Remark 2.2 The following are weakening-free fuzzy logics extending $\mathbf{UL}_\Delta$:

- Involutive UL_Δ $\mathbf{IUL}_\Delta$ is $\mathbf{UL}_\Delta$ plus (DNE) $\sim\sim\phi \to \phi$.
- Idempotent UL_Δ $\mathbf{UML}_\Delta$ is $\mathbf{UL}_\Delta$ plus (ID) $(\phi \,\&\, \phi) \leftrightarrow \phi$.
- Involutive UML_Δ $\mathbf{IUML}_\Delta$ is $\mathbf{IUL}_\Delta$ plus (ID) and (FP) $\mathbf{t} \leftrightarrow \mathbf{f}$.

By eliminating A13 to A21 from the systems $\mathbf{UL}_\Delta$, $\mathbf{IUL}_\Delta$, $\mathbf{UML}_\Delta$, and $\mathbf{IUML}_\Delta$, we obtain the weakening-free systems **UL**, **IUL** (Involutive uninorm logic), **UML** (Uninorm mingle logic), and **IUML** (Involutive uninorm mingle ligic), respectively, introduced in [7]. Note that the system **MAILL** (Multiplicative additive intuitionistic linear logic) is the **UL** omitting A12.

For easy reference, we let Ls_{Δ} be the set of weakening-free fuzzy logics extending $\mathbf{UL}_{\Delta}$.

An easy computation shows the following.

Proposition 2.3 L_{Δ} ($\in Ls_{\Delta}$) proves:

(1) $(\phi \,\&\, (\psi \,\&\, \chi)) \leftrightarrow ((\phi \,\&\, \psi) \,\&\, \chi)$ (&-associativity, AS)

(2) $(\phi \to (\psi \to \chi)) \to (\psi \to (\phi \to \chi))$ (permutation, RE)

(3) $\phi \vee (\phi \to \mathbf{t})$

(4) $\phi_{t} \vee (\phi \to \mathbf{t})_{t}$

(5) $\phi_{t} \to (\Delta\phi \leftrightarrow \mathbf{t})$

(6) $(\Delta\phi)_{t} \to (\Delta\phi \leftrightarrow \mathbf{t})$

(7) $(\phi \to \mathbf{t})_{t} \to (\Delta\phi \leftrightarrow \phi)$

(8) $\Delta\Delta\phi \leftrightarrow \Delta\phi$

(9) $\Delta(\phi \,\&\, \phi) \to \Delta\phi$ (ΔSDE)

(10) $(\Delta\phi)_{t} \to (\psi_{t} \to \phi)$ (ΔW_{t})

(11) $(\Delta(\phi \to \psi))^{n}_{t} \vee (\Delta(\psi \to \phi))^{n}_{t}$, for each n ($\Delta PL^{n}_{t}$)

(12) $(\Delta(\phi \to \psi))_{t} \vee (\Delta(\psi \to \phi))_{t}$ (ΔPL_{t})

(13) $\Delta(\phi \to \psi) \vee \Delta(\psi \to \phi)$ (ΔPL)

(14) $\Delta\phi \leftrightarrow (\Delta\phi)_{t} \leftrightarrow \phi_{t}$

(15) $\Delta\phi \to (\Delta\psi \to \phi)$ (ΔWb)

(16) $\Delta(\Delta\phi \to (\Delta\psi \to \chi)) \to (\Delta\psi\to(\Delta\phi\to\chi))$ (ΔPM)

(17) $\phi \vdash \Delta\phi$ (necessitation, nec).

Proof: We prove (5) as an example. By A14, we have $\phi_{t} \to \Delta\phi$. Then, since it holds that $\phi \leftrightarrow (\mathbf{t} \to \phi)$, we get $\phi_{t} \to (\mathbf{t} \to$

$\Delta\phi$). Note that A13 ensures that $\mathbf{t} \rightarrow (\Delta\phi \rightarrow \mathbf{t})$. Thus, since $\phi_{\mathbf{t}} \rightarrow \mathbf{t}$, we have $\phi_{\mathbf{t}} \rightarrow (\Delta\phi \rightarrow \mathbf{t})$. Therefore, $\phi_{\mathbf{t}} \rightarrow (\Delta\phi \leftrightarrow \mathbf{t})$.

The proof for the remaining is left to the reader. □

A *theory* over L_Δ ($\in Ls_\Delta$) is a set T of formulas. A *proof* in a theory T over L_Δ is a sequence of formulas whose each member is either an axiom of L_Δ, a member of T, or follows from some preceding members of the sequence using the rules. T $\vdash \phi$, more exactly T $\vdash_{L\Delta} \phi$, means that ϕ is *provable* in T w.r.t. L_Δ, i.e., there is an L_Δ-proof of ϕ in T. The (local) **t**-deduction theorem ((L)$DT_{\mathbf{t}}$) for L_Δ is as follows:

Proposition 2.4 Let T be a theory over L_Δ ($\in Ls_\Delta$), and ϕ, ψ be formulas.

(i) ($LDT_{\mathbf{t}}$) $T \cup \{\phi\} \vdash_{L\Delta} \psi$ iff there is n such that $T \vdash_{L\Delta} (\phi)^n_{\mathbf{t}} \rightarrow \psi$.

(ii) ($DT_{\mathbf{t}}$) For L_Δ with (ID), $T \cup \{\phi\} \vdash_{L\Delta} \psi$ iff $T \vdash_{L\Delta} (\phi)_{\mathbf{t}} \rightarrow \psi$.

Proof: For (i), see [10]. (ii) is the Enthymematic Deduction Theorem (see [9]). □

A theory T is *inconsistent* if T $\vdash$ **F**; otherwise it is *consistent*.

For convenience, "~", "Δ", "∧", "∨", and "→" are used ambiguously as propositional connectives and as algebraic operators, but context should make their meaning clear.

3. Semantics

Suitable algebraic structures for L_Δ ($\in Ls_\Delta$) are obtained as a subvariety of the variety of commutative residuated lattices (in the sense of [5]) expanded by a unary operation Δ satisfying the conditions below.

Definition 3.1 (i) (MAILL-algebra) A *pointed bounded commutative residuated lattice* is a structure **A** = (A, ⊤, ⊥, t, f, ∧, ∨, *, →) such that:

(Ⅰ) (A, ⊤, ⊥, ∧, ∨) is a bounded lattice with top element ⊤ and bottom element ⊥.

(Ⅱ) (A, *, t) satisfies for some t and for all x, y, z ∈ A,

(a) x * y = y * x (commutativity)

(b) t * x = x (identity)

(c) x * (y * z) = (x * y) * z (associativity).

(Ⅲ) y ≤ x→z iff x * y ≤ z, for all x, y, z ∈ A (residuation).

(ii) (L-algebra) A *UL-algebra* is an MAILL-algebra satisfying the condition: for all x, y, (pl_t) $t \leq (x \rightarrow y)_t \vee (y \rightarrow x)_t$; an *IUL-algebra* is a UL-algebra satisfying the condition: for all x, (dn) (x → f) → f = x; a *UML-algebra* is a UL-algebra satisfying the condition: for all x, (id) (x * x) = x; an *IUML-algebra* is a UML-algebra satisfying (dn) and the condition: (fp) f = t. By an *L-algebra*, we ambiguously express any of these algebras.

Notice that the class of pointed commutative residuated lattices characterizes **MAILL**. Thus we call such residuated lattices *MAILL-algebras*.

Definition 3.2 (L_Δ-algebra) Let ~x be x → f. An *L_Δ-algebra* is an L-algebra expanded by a unary operation Δ satisfying:

(Ⅳ) For all $x, y \in A$,

(a) $\Delta x \leq t$

(b) $x_t \leq \Delta x$

(c) $(x \to t) \leq (x \to \Delta x)$

(d) $t \leq \Delta(x \vee y) \to (\Delta x \vee \Delta y)$

(e) $t \leq (\Delta x \wedge \Delta y) \to \Delta(x \wedge y)$

(f) $t \leq \Delta x \to x$

(g) $t \leq \Delta x \to \Delta\Delta x$

(h) $t \leq \Delta(x \to y) \to (\Delta x \to \Delta y)$

(i) $t \leq \Delta(x * x) \to (\Delta x * \Delta x)$.

An MAILL-algebra is said to be *linearly ordered* if the ordering of its algebra is linear, i.e., $x \leq y$ or $y \leq x$ (equivalently, $x \wedge y = x$ or $x \wedge y = y$) for each pair x, y.

Definition 3.3 (Evaluation) Let $\mathcal{A}$ be an L_Δ-algebra. An *$\mathcal{A}$-evaluation* is a function $v : FOR \to \mathcal{A}$ satisfying: $v(\phi \to \psi) = v(\phi) \to v(\psi)$, $v(\phi \wedge \psi) = v(\phi) \wedge v(\psi)$, $v(\phi \vee \psi) = v(\phi) \vee v(\psi)$, $v(\phi \,\&\, \psi) = v(\phi) * v(\psi)$, $v(\Delta\phi) = \Delta v(\phi)$, $v(\mathbf{F}) = \bot$, $v(\mathbf{f}) = f$, (and hence $v(\sim\phi) = \sim v(\phi)$, $v(\mathbf{T}) = \top$, and $v(\mathbf{t}) = t$).

Definition 3.4 Let $\mathcal{A}$ be an L_Δ-algebra, T a theory, ϕ a formula, and K a class of L_Δ-algebras.

(i) (Tautology) ϕ is a *t-tautology* in $\mathcal{A}$, briefly an *$\mathcal{A}$-tautology* (or *$\mathcal{A}$-valid*), if $v(\phi) \geq t$ for each $\mathcal{A}$-evaluation v.

(ii) (Model) An $\mathcal{A}$-evaluation v is an *$\mathcal{A}$-model* of T if $v(\phi) \geq t$ for each $\phi \in T$. By *Mod(T, $\mathcal{A}$)*, we denote the class of $\mathcal{A}$-models of T.

(iii) (Semantic consequence) ϕ is a *semantic consequence* of T w.r.t. K, denoting by $T \vDash_{\mathsf{K}} \phi$, if $Mod(T, \mathcal{A}) = Mod(T \cup \{\phi\}, \mathcal{A})$ for each $\mathcal{A} \in \mathsf{K}$.

Definition 3.5 (**L_Δ**-algebra) Let $\mathcal{A}$, T, and ϕ be as in Definition 3.4. $\mathcal{A}$ is an ***L_Δ*-*algebra*** iff whenever ϕ is L_Δ-provable in T (i.e. $T \vdash_{L\Delta} \phi$), it is a semantic consequence of T w.r.t. the set $\{\mathcal{A}\}$ (i.e. $T \vDash_{\{A\}} \phi$), $\mathcal{A}$ an L_Δ-algebra. By *$MOD^{(l)}(L_\Delta)$*, we denote the class of (linearly ordered) **L_Δ**-algebras. Finally, we write $T \vDash^{(l)}_{L\Delta} \phi$ in place of $T \vDash_{MOD^{(l)}(L\Delta)} \phi$.

Note that since each condition for the L_Δ-algebra has the form of an equation or can be defined in an equation, it can be ensured that the class of all L_Δ-algebras is a variety.

Before showing the completeness of L_Δ, we note that the truth function for the connective Δ can be given as follows:

$$(\Delta) \qquad \Delta x = \begin{cases} t & \text{if } t \leq x \\ x & \text{otherwise.} \end{cases}$$

A18, Proposition 2.3 (4), (5), and (7) ensure that (Δ) holds in linearly ordered L_Δ-algebras. If t = ⊤, then (Δ) on [0, 1] is the same as the below.

(Δ') $\Delta x = \top$ if $\top = x$
x otherwise.

That is, Δx is the same as x.

Remark 3.6 Since (Δ') is not the same as (BP) in Section 1, it can be ensured that (Δ) can be regarded neither as the Baaz's projection nor as its generalization. But, note that Baaz's Delta projection has the conditions corresponding to A16 to A20, and so (Δ) satisfies many properties, which the Baaz's projection has. In this sense, we can regard (Δ) as a variant of the Baaz's projection.

We first show that classes of provably equivalent formulas form an L_Δ-algebra. Let T be a fixed theory over L_Δ. For each formula φ, let $[\phi]_T$ be the set of all formulas ψ such that $T \vdash_{L\Delta} \phi \leftrightarrow \psi$ (formulas T-provably equivalent to φ). A_T is the set of all the classes $[\phi]_T$. We define that $[\phi]_T \rightarrow [\psi]_T = [\phi \rightarrow \psi]_T$, $[\phi]_T * [\psi]_T = [\phi \,\&\, \psi]_T$, $[\phi]_T \wedge [\psi]_T = [\phi \wedge \psi]_T$, $[\phi]_T \vee [\psi]_T = [\phi \vee \psi]_T$, $\Delta[\phi]_T = [\Delta\phi]_T$, $\bot = [\mathbf{F}]_T$, $\top = [\mathbf{T}]_T$, $t = [\mathbf{t}]_T$, and $f = [\mathbf{f}]_T$. By $\boldsymbol{A_T}$, we denote this algebra.

Proposition 3.7 For T a theory over L_Δ, $\mathbf{A}_T$ is an $\mathbf{L}_\Delta$-algebra.

Proof: Note that A1 to A7 ensure that $\wedge$ and $\vee$ satisfy (I) in Definition 3.1; that AS, A8, A9 ensure that & satisfies (II); that A11 and A12 ensure that (III) and (pl^n_t) hold; that A13 to A21 ensure that (IV) hold. It is obvious that $[\phi]_T \leq [\psi]_T$ iff $T \vdash_{L\Delta} \phi \leftrightarrow (\phi \wedge \psi)$ iff $T \vdash_{L\Delta} \phi \rightarrow \psi$. Finally recall that $\mathbf{A}_T$ is an $\mathbf{L}_\Delta$-algebra iff $T \vdash_{L\Delta} \psi$ implies $T \vDash_{L\Delta} \psi$, and observe that for ϕ in T, since $T \vdash_{L\Delta} \mathbf{t} \rightarrow \phi$, it follows that $[\mathbf{t}]_T \leq [\phi]_T$. Thus it is an $\mathbf{L}_\Delta$-algebra. □

We next note that the nomenclature of the prelinearity condition is explained by the subdirect representation theorem below.

Proposition 3.8 (Cf. [11]) Each L_Δ-algebra is a subdirect product of linearly ordered L_Δ-algebras.

Theorem 3.9 (Strong completeness) Let T be a theory, and ϕ a formula. $T \vdash_{L\Delta} \phi$ iff $T \vDash_{L\Delta} \phi$ iff $T \vDash^1_{L\Delta} \phi$.

Proof: (i) $T \vdash_{L\Delta} \phi$ iff $T \vDash_{L\Delta} \phi$. The left-to-right direction follows from Definition 3.5 and Proposition 3.7. The right-to-left direction is as follows: From Proposition 3.7, we obtain $\mathbf{A}_T \in MOD(L_\Delta)$, and for $\mathbf{A}_T$-evaluation v defined as $v(\psi) = [\psi]_T$, it holds that $v \in Mod(T, \mathbf{A}_T)$. Thus, since from $T \vDash_{L\Delta} \phi$ we obtain $[\phi]_T = v(\phi) \geq t$, $T \vdash_{L\Delta} \mathbf{t} \rightarrow \phi$. Then, since $T \vdash_{L\Delta} \mathbf{t}$, by (mp), we get $T \vdash_{L\Delta} \phi$, as required.

(ii) $T \vDash_{L\Delta} \phi$ iff $T \vDash^1_{L\Delta} \phi$. It follows from Proposition 3.8. □

4. Concluding remark

Here we investigated weakening-free fuzzy logics with the connective Δ, which can be regarded as a variant of the Baaz projection. We introduced the corresponding algebraic semantics and proved their algebraic completeness.

We do not investigate standard completeness, i.e., completeness on the real interval [0, 1], for the systems. This is a problem left in this paper.[1)]

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1) 금번 논문에서 필자는 일부 심사자의 의견을 충분히 반영하지 못하였다. 그 이유는 일차적으로 심사자들의 지적을 다 반영하려면 이 논문에서 추가적으로 설명되어야 할 것들이 너무 많다는 것이다. 부차적으로 심사자들의 일부 요구는 필자의 해당 분야에서는 너무 당연한 것이어서 굳이 논할 필요가 없다는 것이다. 이러한 점들을 반영하여 필자는 이 논문을 수정, 보완하였다. 부족한 부분에 대해서는 심사자들의 너그러운 이해를 바란다.

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ARTICLE ABSTRACTS

Weakening-free fuzzy logics with the connective Δ (II) : a variant of Baaz projection

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양은석은 [12]에서 Baaz 사영과 그것의 일반화로 간주될 수 있는 델타 연결사 Δ에 의해 확대된 약화로부터 자유로운 퍼지 논리들을 연구하였다. 이 논문에서 우리는 Baaz 사영의 많은 성질들을 만족하지만 Baaz 사영으로도 그것의 일반화로도 간주될 수 없다는 의미에서 Baaz 사영의 변형에 해당하는 델타 연결사 Δ에 의해 확대된 약화로부터 자유로운 퍼지 논리들을 연구한다. 이를 위하여 먼저 연결사 Δ를 갖는 몇몇 약화로부터 자유로운 퍼지 논리를 소개한다. 다음으로 그에 상응하는 대수적 구조들을 정의한 후, 관련된 대수적 완전성을 증명한다.

주요어: (준구조) 퍼지 논리, 유니놈 논리, Baaz 사영과 그 변형.