논리연구 22-1(2019) pp. 25-42

Set-theoretic Kripke-style Semantics for Weakly Associative Substructural Fuzzy Logics*

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【Abstract】This paper deals with Kripke-style semantics, which will be called set-theoretic Kripke-style semantics, for weakly associative substructural fuzzy logics. We first recall three weakly associative substructural fuzzy logic systems and then introduce their corresponding Kripke-style semantics. Next, we provide set-theoretic completeness results for them.

【Key Words】(Set-theoretic) Kripke-style semantics, Relational semantics, Fuzzy logic, Substructural logic, Weakly associative logic.

Received: Oct. 30, 2018. Revised: Dec. 18, 2018. Accepted: Dec. 5, 2018.

* I must thank the referees for their helpful comments.

1. Introduction

The aim of this paper is to introduce *set-theoretic* Kripke-style semantics for weakly associative substructural fuzzy logic. For this, we first note that Yang introduced two kinds of (binary) Kripke-style semantics, i.e., algebraic and non-algebraic Kripke-style semantics, for logics with pseudo-Boolean (briefly, pB) and de Morgan (briefly, dM) negations in Yang (2015b), logics with weak-Boolean (briefly, wB) negations, which can be regarded as paraconsistent logics in Yang (2014b, 2015a), and weakening-free non-commutative substructural fuzzy logics in Yang (2016a, 2018b). Recently, Yang (2018a) further introduced *algebraic* Kripke-style semantics for some weakly associative substructural fuzzy logics. However, he did not consider *set-theoretical* semantics for them. Thus, it is not clear whether this semantics works for weakly associative substructural fuzzy logic systems. (Note that as Kripke's works show in Kripke (1963, 1965a, 1965b), this semantics is interesting in the sense that it can treat logics to which algebraic semantics cannot be applied.)

This is a tough question because Kripke-style semantics for well-known substructural core fuzzy logic systems are algebraic, but not set-theoretical. As Yang mentioned in Yang (2014a), after algebraic semantics for t-norm[1)] (based) logics were introduced, their corresponding algebraic Kripke-style semantics have been introduced. For instance, after algebraic semantics for monoidal

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1) T-norms are commutative, associative, increasing, binary functions with identity 1 on the real unit interval [0,1].

t-norm (based) logics were introduced by Esteva and Godo in Esteva & Godo (2001), their corresponding algebraic Kripke-style semantics were introduced in Montagna & Ono (2002), Montagna & Sacchetti (2003; 2004), and Diaconescu & Georgescu (2007).[2)]

These facts give rise to the following question:

- Can we introduce set-theoretical Kripke-style semantics for weakly associative substructural fuzzy logics?

The answer to the question is positive in the sense that we can provide such Kripke-style semantics for the weakly associative substructural fuzzy logics introduced in Yang (2016b, 2018b). For this, first, in Section 2 we recall the wta-monoidal uninorm logic $\mathbf{WA_tMUL}$ and its two axiomatic extensions and then introduce their corresponding Kripke-style semantics. In Section 3, using set-theoretic method, we provide soundness and completeness results for them..

For convenience, we shall adopt the notations and terminology similar to those in Cintula (2006), Metcalfe & Montagna (2007), Montagna & Sacchetti (2003; 2004), and Yang (2012; 2014a; 2016a), and we assume reader familiarity with them (along with results found therein).

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2) See Yang (2018b) for more examples.

2. Weakly associative substructural fuzzy logics and Kripke-style semantics

Here we first briefly recall the systems introduced in Yang (2016b) as preliminaries. Weakly associative substructural fuzzy logics are based on a countable propositional language with formulas *Fm* built inductively as usual from a set of propositional variables *VAR*, binary connectives $\to$, &, $\wedge$, $\vee$, and constants **T**, **F**, **f**, **t**., with defined connectives:

df1. $\neg\phi := \phi \to \mathbf{f}$,
df2. $\phi \leftrightarrow \psi := (\phi \to \psi) \wedge (\psi \to \phi)$.

We may define **t** as $\mathbf{f} \to \mathbf{f}$. We moreover define $\phi^n{}_\mathbf{t}$ as $\phi_\mathbf{t}$ & $\cdots$ & $\phi_\mathbf{t}$, n factors, where $\phi_\mathbf{t} := \phi \wedge \mathbf{t}$. For the rest of this paper, we use the customary notations and terminology, and the axiom systems to provide a consequence relation.

We start with the following axiom schemes and rules for the weak **t**-associative monoidal uninorm logic $\mathbf{WA_tMUL}$ and its two axiomatic extensions.

Definition 2.1 (Yang (2016b))

(i) $\mathbf{WA_tMUL}$ consists of the following axiom schemes and rules:

A1. $\phi \to \phi$ (self-implication, SI)
A2. $(\phi \wedge \psi) \to \phi$, $(\phi \wedge \psi) \to \psi$ ($\wedge$-elimination, $\wedge$-E)
A3. $((\phi\to\psi)\wedge(\phi\to\chi)) \to (\phi\to(\psi\wedge\chi))$ ($\wedge$-introduction, $\wedge$-I)

A4. $\phi \to (\phi \vee \psi)$, $\psi \to (\phi \vee \psi)$ ($\vee$-introduction, $\vee$-I)

A5. $((\phi\to\chi)\wedge(\psi\to\chi)) \to ((\phi\vee\psi)\to\chi)$ ($\vee$-elimination, $\vee$-E)

A6. $\mathbf{F} \to \phi$ (ex falsum quodlibet, EF)

A7. $(\phi \,\&\, \psi) \to (\psi \,\&\, \phi)$ (&-commutativity, &-C)

A8. $\phi \leftrightarrow (\mathbf{t} \to \phi)$ (push and pop, PP)

A9. $\phi \to (\psi \to (\psi \,\&\, \phi))$ (&-adjunction, &-Adj)

A10. $(\phi_t \,\&\, \psi_t) \to (\phi \wedge \psi)$ ($\&\wedge$)

A11. $(\psi \,\&\, (\phi \,\&\, (\phi \to (\psi \to \chi)))) \to \chi$ (residuation, Res')

A12. $(\phi \to ((\phi\&(\phi \to \psi)) \,\&\, (\psi \to \chi))) \to (\phi \to \chi)$ (T')

A13. $((\delta\&\varepsilon)\to(\delta\&(\varepsilon\&(\phi\to\psi)_t)))\vee(\delta'\to(\varepsilon'\to((\varepsilon'\&\delta')\&(\psi\to\phi)_t)))$ (PL)

A14. $(\phi_t\&(\psi_t\&\chi_t)) \leftrightarrow ((\phi_t\&\psi_t)\&\chi_t)$ (weak **t**-associativity, $\mathrm{wAS_t}$)

$\phi \to \psi, \phi \vdash \psi$ (modus ponens, mp)

$\phi \vdash \phi_t$ ($\mathrm{adj_u}$)

$\phi \vdash (\delta \,\&\, \varepsilon) \to (\delta \,\&\, (\varepsilon \,\&\, \phi))$ (α)

$\phi \vdash \delta \to (\varepsilon \to ((\varepsilon \,\&\, \delta) \,\&\, \phi))$ (β).

(ii) The following are weakly associative substructural fuzzy logics that axiomatically extend $\mathbf{WA_tMUL}$:

- **t**-associative (ta-) monoidal uninorm logic $\mathbf{A_tMUL}$ is $\mathbf{WA_tMUL}$ plus

$(\mathrm{AS_t})$ $(\phi \,\&\, (\psi \,\&\, \chi))_t \leftrightarrow ((\phi \,\&\, \psi) \,\&\, \chi)_t$;

$(\mathrm{RE_t})$ $(\phi \to (\psi \to \chi))_t \leftrightarrow ((\phi \,\&\, \psi) \to \chi)_t$;

$(\mathrm{SF_t})$ $(\phi \to \psi)_t \to ((\psi \to \chi) \to (\phi \to \chi))$;

$(\mathrm{PF_t})$ $(\psi \to \chi)_t \to ((\phi \to \psi) \to (\phi \to \chi))$; and

$(\mathrm{MT_t})$ $(\phi \to \psi)_t \to ((\phi \,\&\, \chi) \to (\psi \,\&\, \chi))$.

- Strong ta-monoidal uninorm logic $\mathbf{SA_tMUL}$ is $\mathbf{A_tMUL}$ plus

$(\mathrm{sAS_t})$ $(\phi_t \,\&\, (\psi \,\&\, \chi)) \leftrightarrow ((\phi_t \,\&\, \psi) \,\&\, \chi)$.

For easy reference, we let Ls be a set of weakly associative substructural fuzzy logics defined previously.

Definition 2.2 Ls = {$\mathbf{WA_tMUL}$, $\mathbf{A_tMUL}$, $\mathbf{SA_tMUL}$}.

In L $\in$ Ls, **f** can be defined as $\neg\mathbf{t}$ and vice versa.

A *theory* over L $\in$ Ls is a set T of formulas. A *proof* in a sequence of formulas whose each member is either an axiom of L or a member of T or follows from some preceding members of the sequence using a rule of L. T $\vdash$ ϕ, more exactly T $\vdash_L$ ϕ, means that ϕ is *provable* in T w.r.t. L, i.e., there is an L-proof of ϕ in T. A theory T is *inconsistent* if T $\vdash$ **F**; otherwise it is *consistent*.

The deduction theorem for L is as follows:

Proposition 2.3 Let T be a theory, and ϕ, ψ formulas.

(i) (Cintula, Horčík, & Noguera (2013, 2015)) T $\cup$ {ϕ} $\vdash_L$ ψ iff T $\vdash_L$ $\gamma(\phi) \to \psi$ for some γ $\in$ $\Pi(\mathrm{bDT}^*)$.[3)]

(ii) (Yang (2009)) For L $\in$ {$\mathbf{A_tMUL}$, $\mathbf{SA_tMUL}$}, T $\cup$ {ϕ} $\vdash_L$ ψ iff there is n such that T $\vdash_L$ $\phi^n_{\mathbf{t}} \to \psi$.

For convenience, "$\neg$," "$\wedge$," "$\vee$," and "$\to$" are used ambiguously as propositional connectives and as frame operators, but context should clarify their meanings.

Next we provide Kripke-style semantics for Ls. First, Kripke

3) For γ and $\Pi(\mathrm{bDT}^*)$, see Cintula, Horčík, & Noguera (2013; 2015) and Yang (2015a).

frames are defined as follows.

Definition 2.4 (i) (Kripke frame) A *Kripke frame* is a structure **X** = (X, ⊤, ⊥, t, f, ≤, ∗, →) such that (X, ⊤, ⊥, t, f, ≤, ∗, →) is a linearly ordered pointed bounded commutative rlu-groupoid.[4)] The elements of **X** are called *nodes*.

(ii) (MICAL frame) An *MICAL frame* is a Kripke frame, where ∗ is conjunctive (i.e., $\bot * \top = \bot$) and left-continuous (i.e., whenever $\sup\{x_i : i \in I\}$ exists, $x * \sup\{x_i : i \in I\} = \sup\{x * x_i : i \in I\}$), and so its residuum → is defined as $x \to y := \sup\{z: x * z \leq y\}$ for all $x, y \in X$.

Definition 2.5 (L frame) A *WA$_t$MUL frame* is an MICAL frame satisfying (wAS$_t^{A}$') $x_t * (y_t * z_t) = (x_t * y_t) * z_t$, for all $x, y, z \in A$; an *A$_t$MUL frame* is an MICAL frame satisfying (AS$_t^{A}$') $(x * (y * z))_t = ((x * y) * z)_t$,, for all $x, y, z \in A$; and an *SA$_t$MUL frame* is an A$_t$MUL frame satisfying (sAS$_t^{A}$') $x_t * (y * z) = (x_t * y) * z$, for all $x, y, z \in A$. We call all these frames *L frames*.

Definition 2.4 (ii) ensures that an MICAL frame has a supremum w.r.t. ∗, i.e., for every $x, y \in X$, the set $\{z: x * z \leq y\}$ has the supremum. **X** is said to be *complete* if ≤ is a complete order.

An *evaluation* or *forcing* on a set-theoretic Kripke frame is a

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4) For more detailed interpretation of the notations for rlu-groupoids, see Galatos et al. (2007).

relation $\Vdash$ between nodes and propositional variables, and arbitrary formulas subject to the conditions below: for every propositional variable p,

(AHC) if $x \Vdash p$ and $y \leq x$, then $y \Vdash p$;
(min) $\bot \Vdash p$; and

for arbitrary formulas,

(t) $x \Vdash \mathbf{t}$ iff $x \leq t$;
(f) $x \Vdash \mathbf{f}$ iff $x \leq f$;
($\bot$) $x \Vdash \mathbf{F}$ iff $x = \bot$;
($\wedge$) $x \Vdash \phi \wedge \psi =$ iff $x \Vdash \phi$ and $x \Vdash \psi$;
($\vee$) $x \Vdash \phi \vee \psi$ iff $x \Vdash \phi$ or $x \Vdash \psi$;
(&) $x \Vdash \phi \,\&\, \psi$ iff there are $y, z \in X$ such that $y \Vdash \phi$, $z \Vdash \psi$, and $x \leq y * z$;
($\rightarrow$) $x \Vdash \phi \rightarrow \psi$ iff for all $y \in X$, if $y \Vdash \phi$, then $x * y \Vdash \psi$.

An evaluation or forcing on an L frame is an evaluation or forcing further satisfying that (max) for every atomic sentence p, $\{x : x \Vdash p\}$ has a maximum.

Definition 2.6 (i) (Kripke model) A *Kripke model* is a pair (**X**, $\Vdash$), where **X** is a Kripke frame and $\Vdash$ is a forcing on **X**.
(ii) (L model) An *L model* is a pair (**X**, $\Vdash$), where **X** is an L frame and $\Vdash$ is a forcing on **X**. an L model (**X**, $\Vdash$) is said to

be *complete* if **X** is a complete frame and $\Vdash$ is a forcing on **X**.

Definition 2.7 Given a Kripke model (**X**, $\Vdash$), a node x of **X** and a formula ϕ, we say that *x forces* ϕ to express x $\Vdash$ ϕ. We say that ϕ is *true* in (**X**, $\Vdash$) if t $\Vdash$ ϕ, and that ϕ is *valid* in the frame **X** (expressed by **X** $\vDash$ ϕ) if ϕ is true in (**X**, $\Vdash$) for every forcing $\Vdash$ on **X**.

3. Soundness and completeness for Ls

We first introduce two lemmas, which can be easily proved:

Lemma 3.1 (Cf, Yang (2016b)) (Hereditary Lemma, HL) Let **X** be a Kripke frame. For any sentence ϕ and for all nodes x, y $\in$ **X**, if x $\Vdash$ ϕ and y $\leq$ x, then y $\Vdash$ ϕ.

Lemma 3.2 t $\Vdash$ $\phi \rightarrow \psi$ iff for all x $\in$ **X**, if x $\Vdash$ ϕ, then x $\Vdash$ ψ.

We then provide soundness and completeness results for Ls.

Proposition 3.3 (Soundness, Yang (2018b)) If $\vdash_L \phi$, then ϕ is valid in every L frame.

Now we provide completeness results for Ls using set-theoretical Kripke-style semantics. A theory T is said to be *linear* if, for each pair ϕ, ψ of formulas, we have T $\vdash$ $\phi \rightarrow \psi$

or T $\vdash \psi \to \phi$. By an L-theory, we mean a theory T closed under rules of L. As in relevance logic, by a regular L-theory, we mean an L-theory containing all of the theorems of L. Since we have no use of irregular theories, by an L-theory, we henceforth mean an L-theory containing all of the theorems of L.

Moreover, where T is a linear L-theory, we define the *canonical L frame* determined by T to be a structure $\mathbf{X} = (X_{can}, \top_{can}, \bot_{can}, t_{can}, f_{can}, \leq_{can}, *_{can})$, where $\top_{can} = \{\phi : T \vdash_L \mathbf{T} \to \phi\}$, $\bot_{can} = \{\phi : T \vdash_L \mathbf{F} \to \phi\}$, $t_{can} = T$, $f_{can} = \{\phi : T \vdash_L \mathbf{f} \to \phi\}$, X_{can} is the set of linear L-theories extending t_{can}, $\leq_{can}$ is $\supseteq$ restricted to X_{can}, i.e, x $\leq_{can}$ y iff $\{\phi : x \vdash_L \phi\} \supseteq \{\phi : y \vdash_L \phi\}$, and $*_{can}$ is defined as $x *_{can} y := \{\phi \,\&\, \psi : \text{for some } \phi \in x, \psi \in y\}$ satisfying groupoid properties corresponding to L frames on $(X_{can}, t_{can}, \leq_{can})$. Note that the base t_{can} is constructed as the linear L-theory that excludes nontheorems of L, i.e., excludes ϕ such that $\nvdash_L \phi$. The partial orderedness and the linear orderedness of the canonical L frame depend on $\leq_{can}$ restricted on X_{can}. Then, first, the following is obvious.

Proposition 3.4 A canonical L frame is linearly ordered.

Proof: Since $\leq_{can}$ is an order reversed subset relation, it is obvious that a canonical L frame is partially ordered. For linearly orderedness, suppose for contradiction that neither x $\leq_{can}$ y nor y $\leq_{can}$ x. Then, there exist ϕ, ψ such that $\phi \in y$, $\phi \notin x$, $\psi \in x$, and $\psi \notin y$. Since t_{can} is a linear theory, we have that $\phi \to \psi \in t_{can}$ or $\psi \to \phi \in t_{can}$. Let $\phi \to \psi \in t_{can}$. Then, since ϕ

$\to \psi \in y$, by (mp), we have $\psi \in y$, a contradiction. The case, where $\psi \to \phi \in t_{can}$, is analogous. □

Next, we define a canonical evaluation as follows:

$$\text{(a) } x \Vdash_{can} \phi \text{ iff } \phi \in x.$$

We then consider the following two lemmas.

Lemma 3.5 $t_{can} \Vdash_{can} \phi \to \psi$ iff for all $x \in X_{can}$, if $x \Vdash_{can} \phi$, then $x \Vdash_{can} \psi$.

Proof: By (a), we instead show that $\phi \to \psi \in t_{can}$ iff for all $x \in X_{can}$, if $\phi \in x$, then $\psi \in x$. For the left-to-right direction, we assume $\phi \to \psi \in t_{can}$ and $\phi \in x$, and show $\psi \in x$. By the suppositions and the definition of $*_{can}$, we have that $\phi \& (\phi \to \psi) \in x *_{can} t_{can} = x$. Then, since $(\phi \& (\phi \to \psi)) \to \psi \in t_{can}$ and thus $(\phi \& (\phi \to \psi)) \to \psi \in x$, we also obtain that $\psi \in x$ by (mp). For the right-to-left direction, suppose contrapositively that $\phi \to \psi \notin t_{can}$. Set $x_0 = \{Z : \text{there exists } X \in t_{can} \text{ and } t_{can} \vdash X \to (\phi \to Z)\}$. Clearly, $x_0 \supseteq t_{can}$, $\phi \in x_0$, and $\psi \notin x_0$. (Otherwise, $t_{can} \vdash X \to (\phi \to \psi)$; therefore, $t_{can} \vdash \phi \to \psi$, a contradiction, by (mp), since $t_{can} \vdash X$.)

Then, by the Linear Extension Property of Theorem 12.9 in Cintula, Horčík, & Noguera (2015), we have a linear theory $x \supseteq x_0$ with $\psi \notin x$; therefore $\phi \in x$ but $\psi \notin x$. □

Lemma 3.6 (Canonical Evaluation Lemma) $\Vdash_{can}$ is an evaluation.

Proof: First, consider the conditions for propositional variables.

For (AHC), we need to show that: for every propositional variable p,

$$\text{if } x \Vdash_{can} p \text{ and } y \leq_{can} x, \text{ then } y \Vdash_{can} p.$$

Assume that $x \Vdash_{can} p$ and $y \leq_{can} x$. By (a), we have that $p \in x$ and $x \subseteq y$, and thus $p \in y$; therefore, $y \Vdash_{can} p$.

For (min), we need to show that: for every propositional variable p,

$$\bot_{can} \Vdash_{can} p.$$

By (a), we need to show that $p \in \bot_{can}$. Since $\bot_{can} = \{\phi : T \vdash_L \mathbf{F} \rightarrow \phi\}$, we have that $p \in \bot_{can}$; therefore, $\bot_{can} \Vdash_{can} p$ by (a).

We next consider the conditions for propositional constants **t**, **f**, and **F**.

For (t), we need to show that:

$$x \Vdash_{can} \mathbf{t} \text{ iff } x \leq_{can} t_{can}.$$

Since $t_{can} = T$ and x is a theory extending T, we can ensure that $\mathbf{t} \in x$ iff $x \supseteq t_{can}$; therefore, $x \Vdash_{can} \mathbf{t}$ iff $x \leq_{can} t_{can}$ by (a).

The proof for (f) and ($\bot$) is analogous to that of (t).

Now we consider the conditions for arbitrary formulas.

For ($\wedge$), we need to show

$$x \Vdash_{can} \phi \wedge \psi \text{ iff } x \Vdash_{can} \phi \text{ and } x \Vdash_{can} \psi.$$

By (a), we instead show that $\phi \wedge \psi \in x$ iff $\phi \in x$ and $\psi \in x$. The left-to-right direction follows from ($\wedge$-E) and (mp). The right-to-left direction follows from (adj).

For ($\vee$), we must show

$$x \Vdash_{can} \phi \vee \psi \text{ iff } x \Vdash_{can} \phi \text{ or } x \Vdash_{can} \psi.$$

By (a), we instead show that $\phi \vee \psi \in x$ iff $\phi \in x$ or $\psi \in x$. The left-to-right direction follows from the fact that linear theories are also prime theories in L (see Cintula & Noguera (2011)). The right-to-left direction follows from ($\vee$-I) and (mp).

For (&), we need to show

$$x \Vdash_{can} \phi \,\&\, \psi \text{ iff there are } y, z \in X \text{ such that } y \Vdash_{can} \phi,\ z \Vdash_{can} \psi, \text{ and } x \leq_{can} y *_{can} z.$$

The definition of $*_{can}$ ensures that $\phi \,\&\, \psi \in x$ iff there are $y, z \in X$ such that $\phi \in y$, $\psi \in z$, and $x \leq_{can} y *_{can} z$. Then, by (a), we obtain the claim.

For ($\to$), we need to show

$$x \Vdash_{can} \phi \to \psi \text{ iff for all } y \in X, \text{ if } y \Vdash_{can} \phi, \text{ then } x *_{can} y \Vdash_{can} \psi.$$

By (a), we instead show that $\phi \to \psi \in x$ iff for all $y \in X$, if $\phi \in y$, then $\psi \in x *_{can} y$. For the left-to-right direction, we assume $\phi \to \psi \in x$ and $\phi \in y$, and show $\psi \in x *_{can} y$. The definition of $*_{can}$ ensures $(\phi \to \psi) \,\&\, \phi \in x *_{can} y$. Since $((\phi \to \psi) \,\&\, \phi) \to \psi \in t_{can}$ and thus $((\phi \to \psi) \,\&\, \phi) \to \psi \in x *_{can} y$, by (mp), we have that $\psi \in x *_{can} y$. For the right-to-left direction, suppose contrapositively that $\phi \to \psi \notin x$. As in Lemma 3.5, we can construct a linear theory y such that $\phi \in y$ and $\psi \notin x *_{can} y$. □

Let us call a model **M**, = (**X**, $\Vdash_{can}$) (i.e., (X_{can}, $\top_{can}$, $\bot_{can}$, t_{can}, f_{can}, $\leq_{can}$, $*_{can}$, $\Vdash_{can}$)), for L, an L model. Then, by Lemma 3.6, the canonically defined (**X**, $\Vdash_{can}$) is an L model. Thus, since, by construction, t_{can} excludes our chosen nontheorem ϕ, and the canonical definition of models agrees with membership, we can state that, for each nontheorem ϕ of L, there is an L model in which ϕ is $t_{can} \nvDash_L \phi$. It gives us the weak completeness of L as follows.

Theorem 3.7 (Weak completeness) If $\vDash_L \phi$, then $\vdash_L \phi$.

Furthermore, using Lemma 3.6 and the Linear Extension Property, we can show the strong completeness of L as follows.

Theorem 3.8 (Strong completeness) L is strongly complete w.r.t. the class of all L-frames.

4. Concluding remark

Here we investigated set-theoretic Kripke-style semantics for some weakly associative substructural fuzzy logics. Note that, while Yang provided algebraic semantics for other non-associative substructural fuzzy logics in Yang (2015a, 2016c, 2017a, 2017b). he did not consider set-theoretic Kripke-style semantics for those systems. To provide such semantics for these logics remains a problem to be solved.

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약한 결합 원리를 갖는 준구조 퍼지 논리를 위한 집합 이론적 크립키형 의미론

양 은 석

이 글에서 우리는 (곱 연언 &의) 약한 형식의 결합 원리를 갖는 준구조 퍼지 논리를 위한 집합 이론적 크립키형 의미론을 연구한다. 이를 위하여 먼저 약한 결합 원리를 갖는 세 준구조 퍼지 논리 체계들을 상기한 후 이 체계들에 상응하는 크립키형 의미론을 소개한다. 다음으로 집합 이론적 방식을 이용하여 이 체계들이 완전하다는 것을 보인다.

주요어: (집합 이론적) 크립키형 의미론, 관계 의미론, 퍼지 논리, 약한 결합 원리, 준구조 논리